Review for Midterm 2

Transpose

2.5.19

Four fundamental subspaces

3.2.6, 3.2.11, Quiz 6 (a, b)

Linear independence, basis, dimension, and rank–nullity theorem

Quiz 6 (c), Prove Theorem 4.9, 3.4.24,

Vector spaces

3.1.1, 3.6.6

Projection matrix

4.1.2, 4.1.3

Least squares

4.1.9 (you will not be asked to compute errors)

Gram–Schmidt

4.2.3

Some additional problems

Problem 1. Label each of the following statements as true or false.

(a) The set $\{A \in \mathcal{M}_{n\times n} : A = A^\top\}$ is a subspace of $\mathcal{M}_{n\times n}$.

(b) For any $m \times n$ matrix A, the row space and the column space of A have the same dimension.

(c) If A is a matrix such that $\mathbf{N}(A^\top)$ contains $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, then $\mathbf{C}(A)$ contains $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

(d) The columns of any 10×11 matrix are linearly dependent.

Problem 2. Give an example for each of the following, or explain why none exists.

(a) A vector space which does not have a finite basis.

(b) A 3×4 matrix A such that $\dim \mathbf{R}(A) = \dim \mathbf{N}(A^\top)$.

(c) A matrix A such that $\mathbf{N}(A)$ has basis $\begin{bmatrix}1\\0\\1\end{bmatrix}$ and $\mathbf{N}(A^\top)$ has basis $\begin{bmatrix}1\\-1\\1\end{bmatrix}$.

(d) An orthonormal basis of the orthogonal complement of $\text{Span}((2,0,2,6))$ in $\mathbb{R}^4$.

Problem 3. Let A be the 10×3 matrix

$$\begin{bmatrix}0 & 0 & 1\\2 & 1 & 1\\2 & 1 & 2\\4 & 2 & 3\\6 & 3 & 5\\10 & 5 & 8\\16 & 8 & 13\\26 & 13 & 21\\42 & 21 & 34\\68 & 34 & 55\end{bmatrix}.$$

Each row of A after the second row is equal to the sum of the previous two rows.

(a) Find the dimensions of $\mathbf{R}(A)$, $\mathbf{C}(A)$, $\mathbf{N}(A)$, and $\mathbf{N}(A^\top)$.

(b) Give a basis for $\mathbf{N}(A^\top)$.