Math 401 Problem Set 4 (due February 13, 2026)

Problem 1. Consider the map $*\colon G \times G \to G$ defined by $g * a = ag^{-1}$.

(a) Show that this defines a group action of G on itself.

(b) What are the orbits of this action?

Problem 2. Show that the center of a group G is a normal subgroup of G.

Problem 3. Compute the centralizer of the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ in $\mathrm{GL}_2(\mathbb{R})$.

Problem 4. Find the number of ways to color the edges of a regular 15-gon using a colors, where two colorings are considered the same if one can be obtained from the other through a rotation.

Problem 5. Determine the class equation for the dihedral groups (a) D_4, (b) D_5.

Problem 6. Find all normal subgroups of (a) D_4, (b) D_5.

Problem 7. (Exercise 7.2.2) A group of order 21 contains a conjugacy class $C(x)$ of order 3. What is the order of x in the group?

Problem 8. Approximately how long did you spend on this problem set? (Round to the nearest half-hour.)