Math 401 Problem Set 8 (due Friday, March 20, 2026)

Problem 1. Let I be an ideal of a ring R. Show that $I = R$ if and only if I contains a unit.

Problem 2. Let R be an integral domain. Show that the polynomial ring $R[x]$ is also an integral domain, and determine the units in $R[x]$.

Problem 3. Find all maximal ideals in each of the following rings.

(a) $\mathbb{C}[x, y]/(y - 4, y - x^2)$.

(b) $\mathbb{C}[x, y]/(y - 2x + 1, y - x^2)$.

Problem 4. A *principal ideal domain* (or PID) is an integral domain in which every ideal is principal. Prove that in a PID, every nonzero prime ideal is a maximal ideal.

Problem 5. Determine if the following ideals of $\mathbb{Z}[i]$ are prime ideals.
(a) $(2i + 1)$. (b) (13).

Problem 6. Determine if the following ideals of $\mathbb{Z}[x]$ are prime ideals.
(a) $(7, x^2 + 1)$. (b) (3).

Problem 7. Give an example of a maximal ideal of $\mathbb{Z}[x]$ which contains $(2x^3 + 1)$.

Problem 8. Approximately how long did you spend on this problem set? (Round to the nearest half-hour.)

Bonus problem (not graded). Prove that every nonzero prime ideal of $\mathbb{Z}[\sqrt{2}]$ is a maximal ideal.